**Birla Institute of Technology & Science, Pilani**

**Work-Integrated Learning Programmes Division**

**Second Semester 2023-2024**

**Comprehensive Examination**

**(EC-3 Regular)**

Course No. : SS ZC416

Course Title : MATH FOUNDATION FOR DATA SCIENCE

Nature of Exam : Open Book

Weightage : 40%

No. of Pages = 2

# No. of Questions = 10

Duration : 2 ½ Hours

Date of Exam : 19/05/2024 (FN)

Note:

1. Please follow all the *Instructions to Candidates* given on the cover page of the answer book.
2. All parts of a question should be answered consecutively. Each answer should start from a fresh page.
3. Assumptions made if any, should be stated clearly at the beginning of your answer.
4. Use the concepts of rank and nullity of a matrix to identify redundant samples and correlated variables in the following dataset. [4 marks]

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| X1 | X2 | X3 | X4 | X5 |
| 3 | 1 | 7 | 2 | 13 |
| 2 | -4 | 14 | -1 | -10 |
| 5 | 11 | -7 | 8 | 59 |
| 2 | 5 | -4 | -3 | 39 |

1. (a) For two square matrices , prove or disprove  [1 mark]

(b) Is it possible to draw a parabolic curve of the form  through the points (-2, 20),(1, 5),(3, 25)? Justify. [3 marks]

1. (a) Is the set R of real numbers a vector space with usual scalar multiplication but with addition defined as ? Justify. [1 mark]

(b) Is the set a basis for the set of all square matrices of order 2? Justify. [2 marks]

1. Diagonalize  and hence find . [4 marks]
2. Find the rank-1 approximation of . [3 marks]
3. (a) Discuss the maxima and minima of  [1 mark]

(b) Compute the angle between ,using  [2 marks]

1. (a) Let . If  and , then write in terms of the variables and . [4 marks]

(b) Expand in powers of  and . [3 marks]

1. Using the Hessian matrix, find if  takes a minima at .

[ 4 marks]

1. Using the Gradient descent algorithm, find the minima of . Take  and perform two iterations. [4 marks]
2. Consider the dataset

|  |  |  |  |
| --- | --- | --- | --- |
| X1 | X2 | X3 | X4 |
| 1 | 4 | 7 | 8 |
| 2 | 2 | 8 | 4 |
| 1 | 13 | 1 | 5 |

Find a 3D projection of this data using PCA. (You may use the following information: The maximum eigenvalue of the covariance matrix is 34.55, and the corresponding eigenvector is [-0.08,-0.32,1]T. [4 marks]

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